Polynomial Curve Fitting of the Corneal Profile in 2.2-mm Corneal Incision Phacoemulsification

Jaime Tejedor, MD, PhD; Francisco J. Gutiérrez-Carmona, MD, PhD

ABSTRACT

PURPOSE: To model incisional axis and perpendicular corneal profile pattern changes in 2.2-mm corneal incision phacoemulsification.

METHODS: Sixty-seven eyes of 67 patients were included in this prospective, interventional, before–after paired design study. Power vector components were obtained from keratometry (IOLMaster; Carl Zeiss Meditec, Göttingen, Germany) and topography corneal height data with the Pentacam HR (Oculus Optikgeräte, Wetzlar, Germany) preoperatively and at 6 months postoperatively. Second- to sixth-order curve fitting polynomial functions of the corneal profile in the incisional and perpendicular axes were created using Matlab (The Mathworks, Inc., Natick, MA). Multivariate regression analysis was run to study the influence of potential predictors. Correlation of changes in corneal elevation and corneal radius with astigmatic parameters was also obtained.

RESULTS: Significant changes occurred only in the $J_0$ ($P = .004$) and $M$ ($P = .001$) parameters. $R^2$ was high with all of the fitted polynomials (0.98 to 0.99) and although the smallest root mean square error was obtained with sixth-degree polynomials (0.63 to 1.13), they were more badly conditioned and redundant than quadratic polynomials. Corneal flattening changes were obtained on axis, which was the most frequent pattern ($n = 52, 77\%$), but were significantly larger in the incisional side than the non-incisional side ($P = .001$) and only coupled with perpendicular axis steepening in 23 patients. In the non-incisional side on axis, corneal steepening was a relatively frequent pattern ($n = 22$ patients, 33%). Predictors studied for profile pattern of change were only near significance. Corneal radius of curvature changes were significantly correlated with astigmatic parameters.

CONCLUSIONS: Polynomial curve fitting is adequate for corneal biomechanical modeling of curvature and profile changes in the incisional and perpendicular axes of a 2.2-mm incision for phacoemulsification.

PATIENTS AND METHODS

The study adhered to the tenets of the Declaration of Helsinki and received institutional review board approval (08805 PI). Patients gave their consent for inclusion after having understood and being adequately informed of the nature of the study. In this prospective, interventional, before–after paired design study, patients were included if they had visually significant impairment from cataract (causing difficulty in daily tasks, developed as an adult of at least 40 years, and had a postoperative visual acuity of at least 20/40), no other concurrent ocular disease or history of amblyopia, and no previous surgery for the eye undergoing surgery. Before–after phacoemulsification surgery changes were evaluated so that patients acted as their own controls.

Preoperative examination of the cohort included visual acuity testing using an ETDRS chart, refraction, biomicroscopy, funduscropy, corneal eye slit scanner analysis (Pentacam; Oculus Optikgeräte, Wetzlar, Germany), and IOLMaster (V. 4.07; Carl Zeiss Meditec, Göttingen, Germany) measurements for calculation of intraocular lens (IOL) power and keratometry (SRK/T formula was generally used; < 22.0 mm or > 26.0 mm of axial length [Hoffer Q and Holladay formulas, respectively]). The same evaluation was made at 15 days and 1, 3, and 6 months postoperatively, except for the Pentacam scan, which was only repeated at the last visit.

Phacoemulsification was always performed by the same surgeon (JT), creating a 2.2-mm clear corneal incision with the OZil Intelligent Phaco torsional US Infiniti platform (Alcon Laboratories, Fort Worth, TX). After creating a 4.5- to 5-mm capsulorhexis using the stop-and-chop technique, an Acrysof SA60AT single-piece foldable IOL (Alcon Laboratories) was introduced through an IOL insertion cartridge (Monarch III cartridge; Alcon Laboratories). Incisions were located in the steepest corneal axis (identified by preoperative ink marks at the slit lamp). Only patients with the steepest axis (located at 90° ± 30° or 180° ± 30°) were included and the incision was superior. When the steepest axis was found at 180° ± 30°, the incision was placed in the temporal side for a kerometric cylinder power of less than 0.5 diopters (D) and in the nasal side for values of 0.5 D or greater because induced keratometric changes vary between 0.3 and 0.6 D, and nasal incisions induce larger astigmatic change.3,11

Keratometric refractive data obtained with the IOLMaster were converted from diopters to power vector components, as previously described12: spherical lens power M, Jackson cross cylinder at the 180° axis of power J0, and Jackson cross cylinder at the 45° axis of power J45. Power J45 represents oblique astigmatism.

The projection of the power vector P (interpreted as a measure of the overall blurring effect) onto the astigmatic plane (vector J, which represents the cylinder as a single Jackson cylinder lens) was also derived. Negative cylinder notation was used throughout. In this representation, positive values of J0 indicate with-the-rule astigmatism and negative values indicate against-the-rule astigmatism. We used the IOLMaster for power vector decomposition rather than Pentacam HR keratometry because Pentacam corneal keratometry readings are not generally used in IOL calculations and could have relatively poorer repeatability than automated keratometry,13,14 and because we sought to correlate corneal elevation and radius changes data with conventional automated keratometric measurements.

Preoperative and postoperative Pentacam HR elevation values of the anterior cornea corresponding to the axis comprising the zero elevation point and included in the horizontal incisional meridian (and the vertical incisional meridian in a few cases) were represented versus distance to the corneal apex (uppermost corneal point). A second- to sixth-degree order polynomial function curve fitting15 describing the corneal profile in that axis was derived using Matlab (V. R2011; The MathWorks, Inc., Natick, MA). The same procedure was followed for the corneal meridian located orthogonal to the incision. Preoperative and postoperative corneal profiles and the radius were subsequently compared to evaluate general and local flattening or steepening of corneal curvature in the incisional and orthogonal axis meridians studied. This strategy allows function modeling and calculation of parameters corresponding to the corneal profile in that axis.

STATISTICAL ANALYSIS

Surgically induced astigmatism was calculated by subtracting preoperative power vector components from postoperative power vector components. Paired t tests or Wilcoxon tests were used to compare preoperative and postoperative parameters depending on variable distribution. The resultant mean or median of subtraction values were converted to conventional astigmatism to express induced astigmatism more intuitively.

Corneal profile changes in the incised and orthogonal meridians were initially evaluated by visual inspection of the representation of the polynomial functions. Change was evaluated by subtracting preoperative corneal elevation measured data and corneal radius (derived from height data) at each point from the same postoperative values. A postoperative–preoperative positive radius difference indicates flattening and a negative value indicates steepening of the cornea in that location. Total and mean change in elevation and
radius of curvature were calculated by adding height or radius changes of points included and dividing the total by the number of points included, respectively. These values were obtained for the entire corneal profile and its incisional and non-incisional sides (ie, between the nearest and limbus-measured point and corneal apex zero on the side where the incision was made, and between the apex and nearest to the limbus point on the opposite side). The perpendicular-to-incisional meridian was also measured. Elevation data were used in addition to radius of curvature because they were directly measured by the device and not subsequently derived.

We used classic keratometric quantitative evaluation of changes in corneal contours as reference to evaluate and correlate the fitting technique. Spearman correlation coefficient of change in corneal height and radius in the incisional (incisional and non-incisional side considered separately) and perpendicular-to-incisional meridians was calculated with change in keratometric astigmatic power vector components. We studied potential candidate variables as predictive factors for steepening or flattening responses of the incised and orthogonal meridians using logistic regression (enter/stepwise strategy).

### RESULTS

Sixty-seven patients were included in the study, with a mean age of 69.47 years (range: 46 to 85 years). Mean preoperative keratometric cylinder was -0.86 D (range: -0.12 to -2.79 D) and mean preoperative keratometric steepest axis after transformation from radians was located at 85.94° (range: 1.71° to 178.76°); the corresponding postoperative values were -0.76 D (range: -0.24 to -2.88 D) and 76.77° (range: 10.88° to 175.89°), respectively.

### ASTIGMATISM INDUCED BY 2.2-MM CORNEAL INCISION

Preoperative and postoperative mean (range) values of astigmatic parameters are given in Table 1. Because several variables (J₀, J₄₅, and J) did not follow a normal distribution, we used non-parametric statistics for comparisons (Wilcoxon signed-rank test). Significant changes occurred in the J₀ (P = .004) and M (P = .001) parameters as a result of surgery, but not in J (P = .77) and J₄₅ (P = 0.10). Mean corneal surgically induced refractive change was +0.19 D, -0.22 D × 78.9° (approximately 79°).

### POLYNOMIAL FUNCTION CURVE FITTING

A second- to sixth-order polynomial function was fitted using height data obtained with the Pentacam in the 180° meridian, including the zero elevation apex value and in the orthogonal meridian. Fitted polynomials of the quadratic to sixth degree yielded high R² values (0.98 to 0.99, P < .01).

Root mean square error is a scale-dependent measure of accuracy (goodness of fit). Root mean square error of quadratic (range: 8.65 to 40.85) polynomials was similar to that of cubic (range: 10.25 to 29.75) polynomial fitting, but it was larger than fourth- (range: 2.67 to 4.89), fifth- (range: 2.63 to 4.88), and sixth-degree (range: 0.63 to 1.13) polynomial fit (Figure 1). However, because a higher degree may result in a more badly conditioned polynomial, increased redundancy, and more difficult mathematical handling, we chose preoperative and postoperative average polynomials of lower degrees within a comparable goodness of fit (ie, quadratic [goodness of fit comparable to cubic] and fourth- [comparable to fifth] and sixth-degree fitting polynomials) (Figures A-C, available in the online version of this article). Preoperative and postoperative on-axis and perpendicular-to-incisinal axis average fitting polynomials are shown.

Table 2 summarizes total and average change in corneal height and the corresponding change in corneal radius induced by surgery on axis. Corneal flattening changes were obtained. Positive radius changes were detected in the incisional and non-incisional sides of the studied meridian, but they were significantly larger in the incisional side for total (P = .002) and average (P = .001) changes. On-axis corneal flattening occurred in 52 patients (77%), with coupled steepening of the perpendicular meridian in 23. Corneal flattening in the incisional side occurred in 55 patients (82%) and 17 had steepening in the non-incisional side. However, in the non-incisional side, corneal flattening occurred in 45 patients (67%), with 7 experiencing steepening of the incisional side. Therefore, corneal steepening was a relatively frequent pattern in the non-incisional side, occurring in 22 patients (33%); 17 patients had incisional-side flattening.

In the orthogonal meridian, steepening occurred in 33 patients (49%), whereas 34 patients (51%) experi-

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### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Preoperative, Mean (Range)</th>
<th>Postoperative, Mean (Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (D)</td>
<td>43.66 (39.87 to 48.29)</td>
<td>43.95 (39.99 to 48.39)</td>
</tr>
<tr>
<td>J₀ (D)</td>
<td>0.13 (-1.27 to 1.10)</td>
<td>-0.02 (-1.36 to 0.82)</td>
</tr>
<tr>
<td>J₄₅ (D)</td>
<td>0.04 (-0.76 to 1.24)</td>
<td>0.10 (-0.74 to 1.40)</td>
</tr>
<tr>
<td>J (D)</td>
<td>0.43 (0.06 to 1.40)</td>
<td>0.38 (0.12 to 1.44)</td>
</tr>
</tbody>
</table>

D = diopters

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enced flattening in the corneal meridian (ie, more than expected by a theoretical 1:1 coupling ratio because 52 patients experienced flattening of the incised meridian). In the group that showed flattening at 90°, only 6 patients had corresponding steepening in the 180° incisional meridian and 28 patients experienced flattening. Of the 33 patients with steepening at 90°, 23 had corresponding flattening in the 180° meridian and 10 experienced steepening (Table A, available in the online version of this article).

**INDUCTION OF ON-AXIS PARADOXICAL STEEPENING**

In a total of 8 patients, unexpected steepening of the on-axis corneal 180° meridian was detected. Accompanying steepening of the incisional side occurred in 5 of these patients (3 had flattening in the non-incisional side) and the remaining 3 experienced flattening (all of them had steepening in the non-incisional side). Only 2 of the 8 patients had flattening in the corresponding orthogonal corneal meridian, whereas 5 showed a steepening pattern and no changes were detected in the remaining case. Steepening also occurred in 7 additional patients, but was expected after creating the superior incision because the corneal steepest axis was located at 90° ± 30° and thus no paradoxical steepening of the 180° meridian occurred in superior incisions.

Variables studied as potential predictive factors of steepening response were preoperative corneal vector components (J, J0, J45, and M), preoperative cylinder, age, central corneal thickness, and peripheral corneal thickness in the incised meridian on the side of the incision. The only candidate variable entered in the model

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**Figure 1.** Sample of fitting polynomials of the quadratic to sixth degree to the corneal profile in case 25 (vertical meridian). $R^2 = 0.98$ to 0.99 in all fittings. (A) Root mean square error was 12.48, 11.25, 2.97, 2.76, and 0.64 in quadratic, cubic, and fourth-, fifth-, and sixth-degree polynomial fittings, respectively. (B) Diagram represents the coordinate system used.

### Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Corneal Height, Mean (Range) (µm)</th>
<th>Corneal Radius of Curvature, Mean (Range) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On axis</td>
<td>-522 (-12,925 to 853)</td>
<td>3.57 (-12.90 to 120.75)</td>
</tr>
<tr>
<td>Perpendicular axis</td>
<td>-96 (-1,413 to 4,454)</td>
<td>0.36 (-23.38 to 14.85)</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>On axis</td>
<td>-4.98 (-123.1 to 8.12)</td>
<td>0.03 (-0.12 to 1.15)</td>
</tr>
<tr>
<td>Perpendicular axis</td>
<td>-0.97 (-14.57 to 57.84)</td>
<td>0.003 (-0.22 to 0.17)</td>
</tr>
<tr>
<td>Total added (incisional side on axis)</td>
<td>-417 (-14,207 to 6,957)</td>
<td>2.66 (-18.20 to 107)</td>
</tr>
<tr>
<td>Average (incisional side on axis)</td>
<td>-7.69 (-236.78 to 117.92)</td>
<td>0.05 (-0.34 to 1.78)</td>
</tr>
<tr>
<td>Total added (non-incisional side on axis)</td>
<td>-82 (-6,467 to 5,603)</td>
<td>1.26 (-5.42 to 36.41)</td>
</tr>
<tr>
<td>Average (non-incisional side on axis)</td>
<td>-1.47 (-129.34 to 98.30)</td>
<td>0.02 (-0.10 to 0.67)</td>
</tr>
</tbody>
</table>

*Total = added height or radius changes of points included.
Average = total/number of points included.
approaching significance was oblique astigmatism, with the likelihood in favor of steepening increasing for an estimated progressively steeper oblique axis ($P = .08$).

**Correlation of Corneal Radius of Curvature and Height Changes With Astigmatic Parameters**

Correlation of total and average radius of curvature change in the incisional axis was significant and moderate with corneal astigmatic change power vector components, particularly with parameter $J_0$ ($P < .01$, $r = -.45$). Radius of curvature change in the incisional side was also significantly correlated with change in $J_0$ ($P = .002$, $r = -.38$). However, height changes in the incisional side were less strongly correlated with changes in the corneal astigmatism parameters of $J$ ($r = -.26$, $P = .03$), $J_0$ ($r = 0.26$, $P = .031$), and $J_{45}$ ($r = 0.28$, $P = .03$).

**DISCUSSION**

We studied corneal elevation and radius of curvature changes in the incised and perpendicular meridians induced by 2.2-mm corneal incisions for cataract surgery using Scheimpflug scan technology and fitted a second- to sixth-degree polynomial curve to the corneal profile. Astigmatic changes observed are in agreement with those reported by Masket et al.,$^4$ who described a mean change of algebraic keratometric astigmatism of $0.10 \pm 0.08$ D and a vector method change of $0.35 \pm 0.21$ D (aggregate analysis $0.12$ D @ 154, standard deviation: $0.28$) for the 2.2-mm micro-coaxial incision. Other authors have found surgically induced astigmatisms of $0.31 \pm 0.18$,$^5 0.4 \pm 0.2$, $^6 0.64 \pm 0.55$, $^7 0.35,^9$ and $0.48 \pm 0.42$ D$^{11}$ for 2.2-mm micro-coaxial incisions.

Because quantitative evaluation of changes in corneal contour induced by surgery is usually obtained from keratometry (which is based only on points in the central cornea and measures the radius of curvature of a small portion of the central cornea), we used a polynomial curve fitting procedure that incorporates data of the whole corneal meridian. Curve fitting is a process of finding a function that can be used to model data and polynomials are expressions frequently used for problem solving and modeling. We have analyzed how polynomials of different degrees fit the same set of data points by writing a simple program and interactively analyzing data that are displayed in the figure window. A second-degree polynomial could model the corneal profile. Goodness of fit is better with higher degree polynomials, but they are more badly conditioned and increase redundancy. Thus, a quadratic polynomial (despite it modeling the corneal profile with less accuracy) is much easier to handle for further calculations than higher degree polynomials. Using a polynomial function curve fitting strategy, we can derive instantaneous radius of curvature, quantify relative flattening or steepening, and correlate these patterns with power vector decomposition parameters of astigmatism changes. The main advantage of polynomial curve fitting of the corneal profile is that it allows obtaining the average polynomial of several cases and that this polynomial incorporates data of a much greater portion of the cornea than classic keratometric quantitative description. With this methodology, quantitative data (height and radius of curvature) provided by modern corneal topography devices are incorporated with the analysis and changes induced by surgery may be analyzed.

Corneal height and curvature changes occur on axis as a consequence of minimal 2.2-mm clear corneal incisions. On average, corneal flattening occurs in the central cornea of the on-axis meridian, as inferred by previous astigmatic vector analysis work.$^4-11$ We have found that the corresponding incisional side of the incised corneal meridian also experiences flattening in general (with exceptions), but in the non-incisional side. Although steepening is not as frequent as flattening, it is a relatively frequently observed response. Using the criterion of negative radius of curvature difference, coupled steepening of the corresponding perpendicular meridian occurred in less than half of these patients and average change was of slight flattening (to a much lesser extent than in the incisional axis). Therefore, the classic coupling pattern is frequently not observed when data from the studied corneal meridian is incorporated to radius change calculations.

A pattern of interest is the occurrence of steepening in the incised corneal meridian considered as a whole (observed in 8 patients, which was almost 12% of the total cases). The reason for such an apparently paradoxical pattern is not clear. Potential predictive factors analyzed by logistic regression did not yield a conclusive result. The only variable approaching significance was the oblique astigmatism parameter. Apparently, a steeper oblique axis could favor steepening or prevent flattening in the incised horizontal meridian, but significance was not reached with the current data. In 5 of the 8 patients, steepening was observed in the perpendicular meridian (ie, typical coupled corneal flattening was not observed). We hypothesize that biomechanical characteristics of these corneas could frequently lead to a steepening response pattern in both the incised and perpendicular axes, and it could be favored by a steeper oblique corneal meridian.

According to our findings, corneal radius of curvature changes on axis (particularly in the incisional side of the incised meridian) are more strongly and significantly correlated with changes observed in astigmatic
parameters than corneal height changes, which is a reasonable finding because corneal dioptic parameters are derived from corneal radius of curvature and not from elevation data. However, correlations are moderate because changes affecting radius of curvature in all of the on-axis meridians are included, whereas the keratometric astigmatic parameters are affected only by central or paracentral corneal changes.

Results of evaluating corneal height and curvature changes by polynomial function fitting are in agreement with astigmatic changes observed, and this methodology may be useful to model changes of a larger portion of the corneal meridian after cataract surgery and presumably to study biomechanical response of the cornea in astigmatic and other corneal refractive or therapeutic surgery. Changes in the peripheral cornea may help explain the biomechanical response to incisions or keratorefractive surgery in a particular case.

AUTHOR CONTRIBUTIONS
Study concept and design (JT); data collection (JT); analysis and interpretation of data (FJG-C, JT); drafting of the manuscript (FJG-C, JT); critical revision of the manuscript (FJG-C, JT); administrative, technical, or material support (JT); supervision (FJG-C)

REFERENCES
Figure A. Preoperative (blue line) and postoperative (red line) second-degree average polynomials of the 67 patients, representing the (A, B) horizontal and (C, D) vertical corneal meridians (distance to corneal apex in µm). In the horizontal meridian representation, the incisional side is on the left. A detailed view to discriminate the preoperative from postoperative profile is depicted in B and D, corresponding to the horizontal and vertical meridians, respectively. Coordinate system is as depicted in Figure 1B.
Figure B. Preoperative (blue line) and postoperative (red line) fourth-degree average polynomials of the 67 patients, representing the (A, B) horizontal and (C, D) vertical corneal meridians (distance to corneal apex in µm). In the horizontal meridian representation, the incisional side is on the left. A detailed view to discriminate the preoperative from postoperative profile is depicted in B and D, corresponding to the horizontal and vertical meridians, respectively. Coordinate system is as depicted in Figure 1B.
Figure C. Preoperative (blue line) and postoperative (red line) sixth-degree average polynomials of the 67 patients, representing the (A, B) horizontal and (C, D) vertical corneal meridians (distance to corneal apex in µm). In the horizontal meridian representation, the incisional side is on the left. A detailed view to discriminate the preoperative from postoperative profile is depicted in B and D, corresponding to the horizontal and vertical meridians, respectively. Coordinate system is as depicted in Figure 1B.
TABLE A
Distribution of Corneal Incision, Profile, and Pattern of Corneal Profile Change on the 180° and Orthogonal Meridians

<table>
<thead>
<tr>
<th>Location of Corneal Incision</th>
<th>Preoperative Steepest Axis</th>
<th>Pattern of Global Change on the 180° Axis</th>
<th>Pattern of Change in the Orthogonal Meridian</th>
<th>Patterns of Profile Change on the 180° Axis by Side (Incisional Side–Non-incisional Side)*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>180 ± 30°</td>
<td>90 ± 30°</td>
<td>180 ± 30°</td>
<td>90 ± 30°</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>S</td>
<td>F</td>
<td>S</td>
</tr>
<tr>
<td>Total (n = 67)</td>
<td>60</td>
<td>7</td>
<td>52</td>
<td>15</td>
</tr>
<tr>
<td>Horizontal</td>
<td>60</td>
<td>–</td>
<td>52</td>
<td>8</td>
</tr>
<tr>
<td>Nasal</td>
<td>36</td>
<td>–</td>
<td>34</td>
<td>2</td>
</tr>
<tr>
<td>Temporal</td>
<td>24</td>
<td>–</td>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td>Superior</td>
<td>–</td>
<td>7</td>
<td>–</td>
<td>7</td>
</tr>
</tbody>
</table>

*In superior incisions, the 180° meridian side closer to the incision is considered the “incisional side.”

F = flattening; S = steepening